Derivative of Sigmoid function:

$$\sigma(\theta|x) = \frac{1}{1+e^{-\theta x}}$$

$$\frac{dy}{dx} \Longrightarrow y = \frac{u(x)}{v(x)} = \frac{vu' - uv'}{v^2}$$

$$\frac{d\sigma}{d\theta} = \frac{(1+e^{-\theta x})(0) - 1(e^{-\theta x})(-x)}{(1+e^{-\theta x})^2}$$

$$\frac{d\sigma}{d\theta} = \frac{xe^{-\theta x}}{(1+e^{-\theta x})^2} \longrightarrow \frac{x \cdot e^{-\theta x}}{(1+e^{-\theta x})^2}$$

$$\frac{d\sigma}{d\theta} = \frac{1}{1+e^{-\theta x}} \left(\frac{e^{-\theta x}}{1+e^{-\theta x}} \right) (x)$$

$$= x \cdot \frac{1}{1+e^{-\theta x}} \cdot \frac{1+e^{-\theta x} - 1}{1+e^{-\theta x}}$$

$$x \cdot \frac{1}{1+e^{-\theta x}} \cdot \left(\frac{1+e^{-\theta x}}{1+e^{-\theta x}} - \frac{1}{1+e^{-\theta x}} \right)$$

$$x \cdot \sigma(\theta | x)\left(1 - \sigma(\theta | x)\right)$$

$$\frac{\partial \sigma(\theta | x)}{\partial \theta} = x \cdot \sigma(\theta | x)\left(1 - \sigma(\theta | x)\right)$$

$$\frac{\partial \log(h(x^i))}{\partial \theta_j} = \frac{1}{h(x)} \cdot \frac{\partial h(x)}{\partial \theta} \quad (1)$$

$$\frac{\partial \log(1-h(x))}{\partial \theta_j} = \frac{-1}{1-h(x)} \frac{\partial h(x)}{\partial \theta} \quad (2)$$

Para (1):

$$\frac{1}{h(x)} \cdot h(x) \cdot (1-h(x)) \, x = (1-h(x)) x$$

Para (2):

$$\frac{-1}{\cancel{1-h(x)}} \cdot h(x) \left(\cancel{1-h(x)}\right) x = -h(x) x$$

$$\frac{\partial \log(u)}{\partial \theta} = \frac{1}{u} \cdot \frac{\partial u}{\partial \theta}$$

$$y^i(1-h(x))x + (1-y^i)(-h(x)\,x)$$

$$y^i x - \cancel{y^i h(x)}\, x - h(x)\,x + \cancel{y^i h(x)}\, x$$

$$y^i x - h(x)\,x$$

$$[y^i - h(x)]\cancel{x} =$$

Consider that $J(\theta) = -\frac{1}{m}\sum_{i=1}^{m}$

$$-\frac{1}{m}\sum[y^i - h(x)\,x] =$$

$$\frac{1}{m}\sum(h(x) - y^i)\,\cancel{x}$$